EFTs.

$$\mathcal{L}^{IR} = \underline{\mathcal{L}_{renorm.}} + \underline{\mathcal{L}_{Irre}}$$

$$\hookrightarrow (\partial\phi)^2\phi^2 + g_1(\partial\phi)^4 + \cdots$$

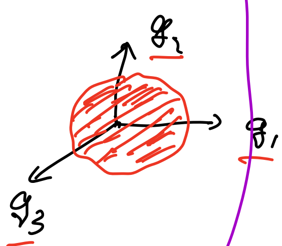
Wilson coefficients

① What is the range of allowed g_is. $\# < g_k < \#$

Leading (WGC) g_k.

② What is the space of EFTs. $\{g_i\}$

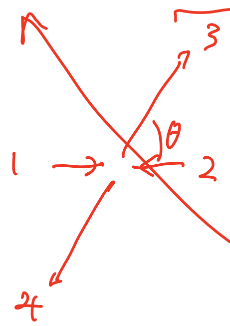
$$\int\sqrt{g}\,R + \frac{1}{4}F^2 + aF^4$$



$$M^{IR}(s,t) = \{\text{Rational terms}\} + \sum_{k,q} g_{k,q}\, s^{k-q}\, t^{q} + \text{log terms}$$

(loop corrections)

dimension of operator

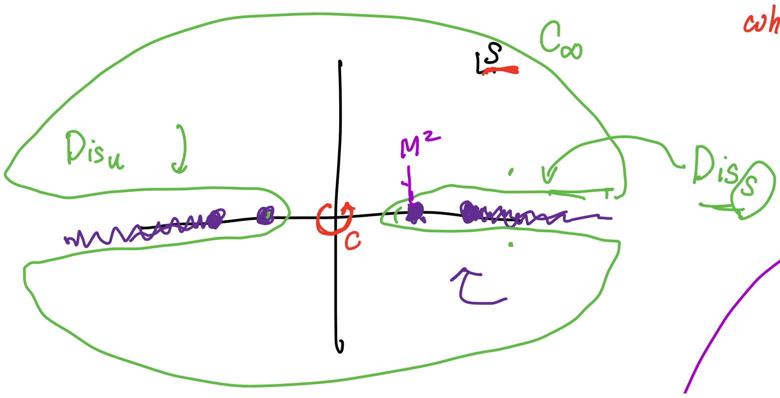


$$t = -\frac{s}{2}(1-\cos\theta)$$

$$k-q \geq 0$$

Assume $M(s,t)$ exists, then $M(s) \equiv M(s,t^*)$

when $|t^*| < \Lambda^2$



$$\mathrm{Dis}[M(s,t^*)] = \sum \rho_{\ell}(s)\, P_\ell(1+\ldots)$$

$t^* = 0$ $\quad \sum \rho_\ell(\,)\, P_\ell(1)$

Assume weakly coupled such that loops can be suppressed

$$g_{k,q} = \left[\frac{1}{q!}\left(\frac{\partial}{\partial t^{\tau}}\right)^{q} \oint \frac{ds}{s^{k-q+1}} M(s) e^{t^{\tau}} \right]\Bigg|_{t^{\tau}=0} \qquad \oint \frac{ds}{s^{2}} \frac{s^{2}}{t}^{t} \qquad \frac{s^{a} \quad a>2}{t=0}$$

$$= \left[\frac{1}{q!}\left(\frac{\partial}{\partial t^{\tau}}\right)^{q}\left(\int^{\infty}_{M^2} \frac{ds'}{s'^{k-q+1}} \mathrm{Dis}(M(s)) + \{u\text{-channel}\}\right)\right]$$

$k-q \geq 2$

$$\boxed{+ \cancel{C_{\infty}}} \leftarrow \frac{s \log s}{s^{k-q+1}} \qquad \frac{s^2}{t} \qquad k-q \geq 2$$

$\frac{s^2}{t}$ $\quad g_{k,q}\, s^{k-q} t^{q}$

$\frac{s^2}{t} \to s^2 t, s^3 t \ldots$

$\frac{s}{t}$ Frossart bound $\boxed{|M(s,t^{\tau})| \underset{\to\infty}{\leq} s \log s}$ $\times$

$$\mathrm{Dis}(M(s)) = \sum_{\ell} \rho_{\ell}(s)\, P_{\ell}\left(1+\frac{2t}{s}\right)$$

$$0 \leq \rho_{\ell}(s) < 2$$

$$\boxed{g_{k,q} = \frac{1}{q!}\left(\frac{\partial}{\partial t}\right)^{q} \int_{M^2}^{\infty} \frac{ds'}{s'^{\,k-q+1}} \rho_{\ell}(s') P_{\ell}\left(1+\frac{2t}{s'}\right) + \{u\}}$$

ρ_{ℓ}: spinning spectral function

$t^{\tau}=0 \quad g_{k,0} = 0$ for odd k.

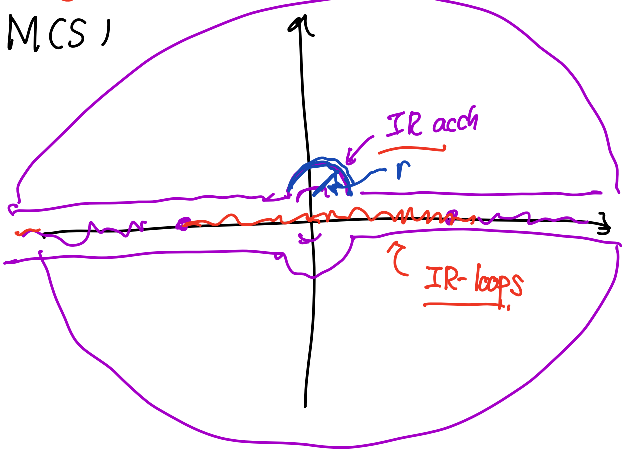
$$g_{k,0} = 2\int_{M^2}^{\infty} \frac{ds'}{s'^{\,k+1}} \underset{>0}{\rho_{\ell}(s')} \underset{1}{P_{\ell}(1)} \qquad \text{even } k$$

$$\boxed{g_{k0} > 0}$$

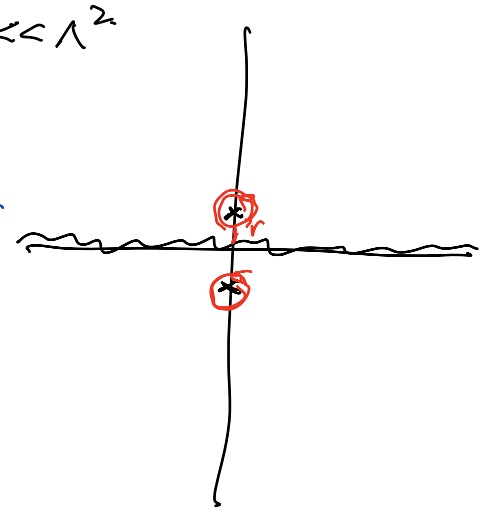
Caveat (1)

$M(s)$

$t^* \ll \Lambda^2$



or



Caveat 2 : massless poles $\quad M(s,t)\big|_{t=t^*} \quad t^* \ll \Lambda^2$

$t^* \to 0$

$$M^{IR}(s,t^*)\big|_{t^*\to 0} = \frac{g^2}{t^*} + \sum g_{kq} \ldots$$

impact parameter

Dis is not converging for $\boxed{g_{k,q},\ k-q=2}$

is converging $\quad k-q>2$

Side comment

$$\mathcal{L}^{IR}(\phi) = \partial\phi\partial\phi + a(\partial\phi)^4$$

$$\phi_B = c_\mu x^\mu \qquad \phi_B = e^{c_\mu x^\mu}$$

$$\mathcal{L}^{IR}(\phi_B + h) \qquad \Box h + a\, c^\mu \partial_\mu c^\nu \partial_\nu h = 0$$

$$\boxed{g_{k,q} = \frac{1}{q!}\left(\frac{\partial}{\partial t}\right)^q \int_{M^2}^{\infty} \frac{ds'}{s'^{\,k-q+1}}\, \rho_\ell(s')\, P_\ell\!\left(1+\frac{2t}{s'}\right) + \{u\}}$$

spinning spectral function

$g_{k,q}$ lives in the convex hull of product moment curves.

Step1: define $z = \frac{M^2}{s}$

$$\int_{M^2}^{\infty} \frac{ds'}{s^{k-q+1}} = \frac{1}{(M^2)^{k-q}} \int_0^1 dz\, z^{k-q-1}$$

Step2

$\xi = \frac{t}{s}$ $\quad P_\ell(1+2\xi) = \sum_{q=1}^{\infty} v_{\ell,q}\, \xi^q$

$J = \ell(\ell+1)$ $\quad v_{\ell,q} = \frac{\prod_{a=1}^{q} (J^2 - a(a-1))}{(q!)^2}$

$$\rightarrow g_{k,q} = \frac{1}{(M^2)^k} \int dz\, z^k \sum_\ell v_{\ell q}\, \rho_\ell(z)$$

$$a_{kq} = [G]_{qq'} g_{k,q'} = \frac{1}{(M^2)^k} \int_0^1 dz \sum_J \rho_J(z)\, z^k\, J^q$$

$$\boxed{\rho_J(z) \geq 0}$$

Moment curves

$$\begin{bmatrix} a_{20} & & & \\ a_{30} & a_{31} & & \\ a_{40} & a_{41} & a_{42} & \\ a_{50} & a_{51} & a_{52} & a_{53} \end{bmatrix} = \int_0^1 dz \sum_J \rho_J(z) \begin{bmatrix} z^2 \\ z^3 \\ z^4 \\ z^5 \\ \vdots \end{bmatrix} \otimes \begin{bmatrix} J^0 \\ J^1 \\ J^2 \\ J^3 \\ \vdots \end{bmatrix}$$

$$= \left[\int_0^1 dz \sum_J \overset{a}{\rho_J'(z)} \begin{bmatrix} 1 \\ z \\ z^2 \\ z^3 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ J \\ J^2 \\ J^3 \end{bmatrix} \right]$$

Space of consistent EFT

$$g_{k,l} \lesssim \frac{O(1)}{M^{2k}}$$

(N)

(Single) Moment problem :

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \vdots \end{pmatrix} = \int_0^1 dz \; p(z)\, z^k \qquad 2 > \boxed{\rho(z) \geq 0}$$

Criteria: Hankel matrix is definite positive
(Shifted)

$$\begin{bmatrix} a_0 & a_1 & a_2 & \cdots \\ a_1 & a_2 & a_3 & \cdots \\ a_3 & a_4 & a_5 & \cdots \\ & & & \end{bmatrix} \checkmark \qquad \begin{bmatrix} a_1 & a_2 & a_3 & \cdots \\ a_2 & a_3 & a_4 & \cdots \\ \cdot & \cdot & \cdot & \\ & & & \end{bmatrix} \checkmark$$

necessary conditions

Double moment problem

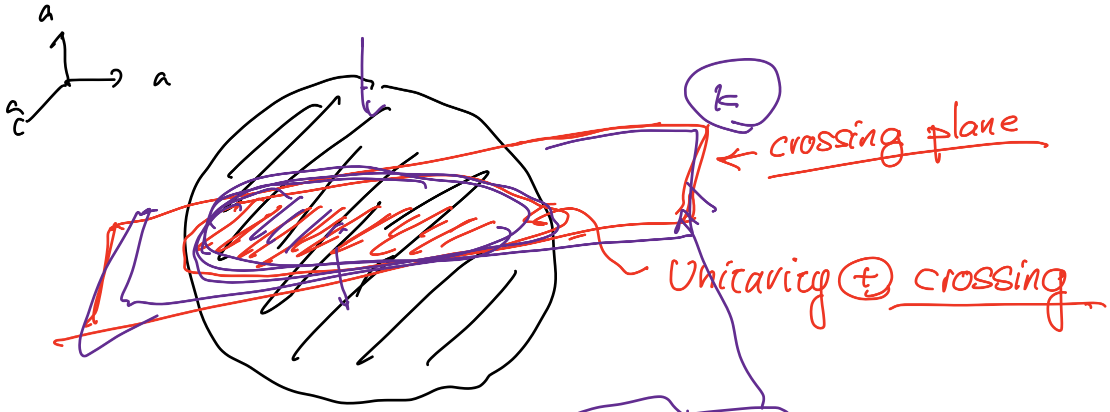
$$\begin{bmatrix} a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \\ \cdot & & \cdot \\ & & \end{bmatrix}$$

generalized hakake

$$\begin{bmatrix} a_{20} & a_{21} & a_{22} & a_{30} & a_{31} & a_{32} & a_{33} & \cdots \\ & & & & & & & \end{bmatrix}$$

symmetric

$$M^{IR} = \sum_{(a,b)} f_{a,b}\, (s^2+t^2+u^2)^a (stu)^b$$



Crossing $g_{4,0} = \# g_{42}$

$k=4 \longrightarrow a(s^2+u^2+t^2)^2$

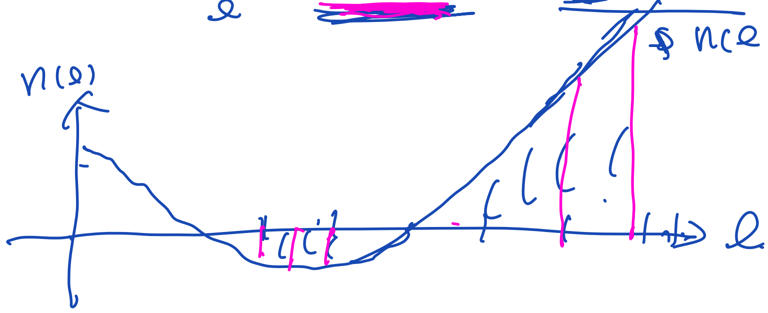
$\{ g_{4,0},\ g_{41},\ g_{42} \}$

$$g_{k,q} = \frac{1}{(M^2)^k} \int dz\, z^k \sum_l v_{lq}\, \rho_l(z)$$

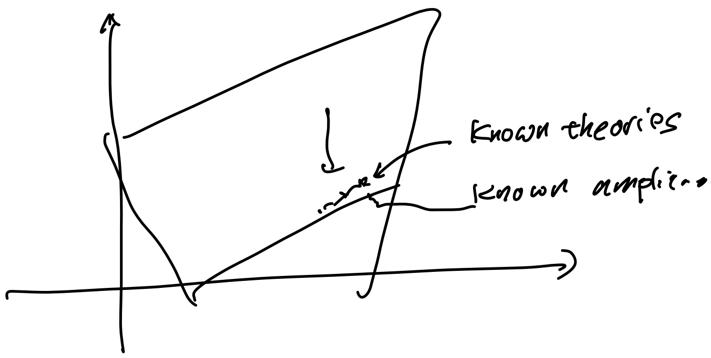
$$g_{4,0} = 8\, g_{42} \Longrightarrow \int dz\, z^4 \rho_l(z) \sum_l (v_{l0} - 8 v_{l2}) = 0$$

$(\# l^2 + \# l + \#)'$

$$\sum_l \langle \rho_l(z) \rangle (\# l^2 + \# l + \#) = 0$$



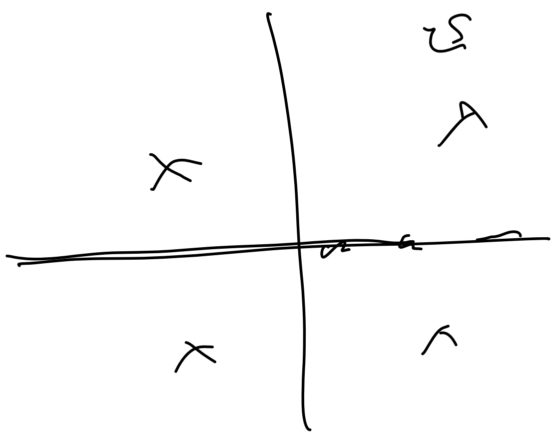
$$\frac{\langle P_\ell(z)\rangle}{\langle P_2(z)\rangle}\Bigg|_{\ell\to\infty} \Rightarrow 0$$



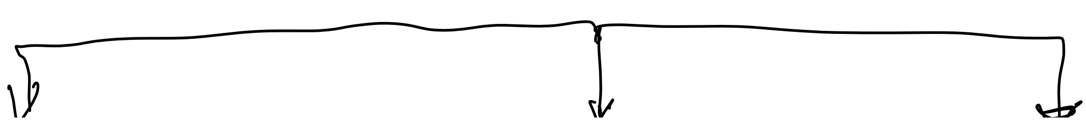
Type II $\qquad R^4 \times \dfrac{\Gamma(-s)\Gamma(-t)\Gamma(-u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)}$

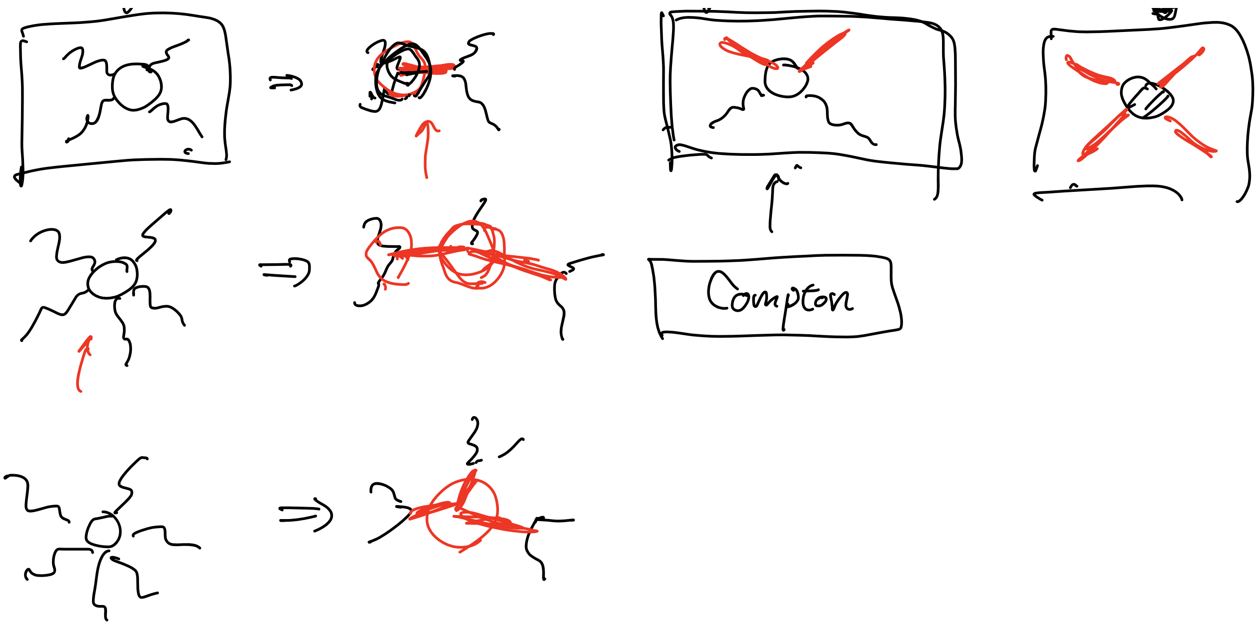
Type II' $\qquad R^4 \times \left(\dfrac{\Gamma\,\Gamma\,\Gamma}{\Gamma\,\Gamma\,\Gamma}\right)(1-\epsilon\, stu)$

$\boxed{0<\epsilon<1}$



$t^{*} \ll M^2$




Compton

$S = 1 + iT$

①

$\mathrm{Im}\,T = T^{\dagger}T = \mathrm{Im}\,a_\ell\, P_\ell(\cos\theta)$

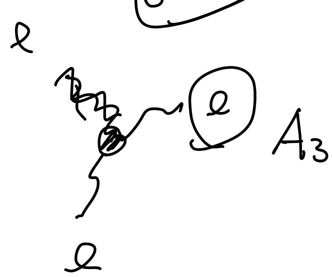
$\sum |x\rangle\langle x|$

$\mathrm{Im}\,a_\ell = \sum_i g_i^* g_i > 0$

$\rho_\ell > 0$

$\int \phi\, \partial^4 \phi \Rightarrow \frac{1}{(p^2)^2}$ $\qquad \frac{1}{s^2}$

L ghosts



$s=0$

$u=-t$

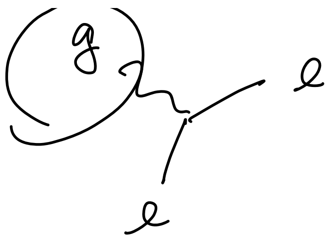
$A_4|_{s=0} =$ 

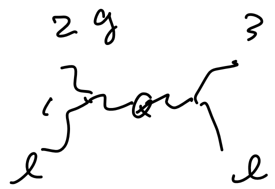
$= \frac{1}{(t)^{\ell-}} \qquad \boxed{\frac{1}{t^2}} = \frac{1}{tu}$

$\ell = 1 \qquad \frac{1}{t^3} =$

$A_4 = \frac{n}{st}$ YM ampl.

$\ell \neq ?$ $\qquad \ell = 2 \quad \frac{n}{stu}$ Gravity





$\to \langle 2|P_1-P_4|3]^{4-2l}$

spurious $l>2$

$\to \dfrac{1}{\langle 2|P_1-P_4|3]}$

$\lambda_2\tilde{\lambda}_3 = \rho$

$\dfrac{1}{(P_1-P_4)\cdot\rho}$